Lecture-1

phases of matter.

- "solids" — Phases with local order parameter
  - gapped
  - gapless (Goldstone mode)
- No local O.P.
  - Trivial [Bosons at integer filling, free fermions] "gases"
  - Non-trivial "liquids"
    - gapped
      - Topologically ordered
        - 1D
          - Kitaev's Majorana chain
        - 2D
          - String net phases
          - QH
        - 3D
          - Discrete gauge theory
          - fractons
        - 4D ...
      - SPT
    - gapless — No quasi particle
      - Non-fermi liquids, gapless spin liquids, Luttinger liquids

Most common phases : Fermi liquids (fermions)
& superfluids (bosons)

How many phases of matter?

one (No)   two ✓   three ✓   infinity ✓

O(2) model

The Hamiltonian is defined as:

$$H = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$

where θ is a compact variable i.e. $\theta \equiv \theta \pm 2\pi$.

In 1d, this model has exponentially decaying correlations at all temperatures $T \neq 0$.

In 3d, there is a low-temperature phase where the O(2) sym. is spontaneously broken: $\langle e^{i\theta(x)} e^{-i\theta(0)} \rangle \to \text{const.} \neq 0$ as $x \to \infty$, for $T < T_c$.

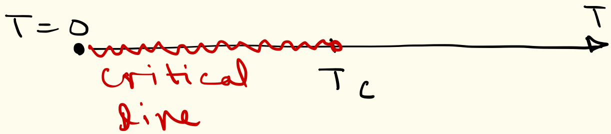
In 2d, this model has a very non-trivial feature that despite the absence of long range order for $T > 0$ (i.e. $\langle e^{i\theta(x)} e^{-i\theta(0)} \rangle \to 0$ as $x \to \infty$) it has two distinct regimes.

For $T < T_c \left(= \frac{\pi J}{2}\right)$, $\langle e^{i\theta(x)} e^{-i\theta(0)} \rangle \sim \frac{1}{|x|^{\frac{T}{2\pi J}}}$

$T > T_c$, $\langle e^{i\theta(x)} e^{-i\theta(0)} \rangle \sim e^{-|x|/\xi}$.

The $T < T_c$ regime should be considered as an infinite number of critical points, since the power-law for correlation f^n changes continuously with T.

Phase diagram:



High-T regime: It is easy to show that at high-T, correlations are exponentially decaying.

$$\langle e^{i\theta(x)} e^{-i\theta(0)} \rangle \propto \mathrm{Tr}\left[e^{i(\theta(x)-\theta(0))} \sum_{n=0}^{\infty} \frac{(-\beta)^n H^n}{n!} \right].$$

Since $\mathrm{Tr}\, e^{i\theta} = 0$, and Hamiltonian only involves nearest-neighbor interactions, the first non-zero contribution is obtained at order $n = d$ where d is the shortest path from 0 to $\vec{x}$. Clearly $d \propto |\vec{x}|$

$$\Rightarrow \langle e^{i\theta(x)} e^{-i\theta(0)} \rangle \sim (\beta J)^{|x|}$$

$$\sim e^{-|x| \log\left(\frac{1}{\beta J}\right)}.$$

Next, we will study the more interesting case of low-T regime.